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A NOTE ON THE SET OF PERIODS FOR CONTINUOUS MAPS OF THE CIRCLE WHICH HAVE DEGREE ONE

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ABSTRACT. The main result of this paper is to complete Misiurewicz's characterization of the set of periods of a continuous map f of the circle with degree one (which depends on the rotation interval of f). As a corollary we obtain a kind of perturbation theorem for maps of the circle of degree one, and a new algorithm to compute the set of periods when the rotation interval is known.

Also, for maps of degree one which have a fixed point, we describe the relationship between the characterizations of the set of periods of Misiurewicz and Block.

1. Notation. We denote by N, Z, Q and R, as usual, the set of positive integers, integers, rational and real numbers, respectively.

Let S^1 be the circle and $C_1(S^1)$ be the set of continuous maps from the circle into itself of degree one. For a map $f \in C_1(S^1)$, $P(f)$ denotes the set of periods of f (from now on, by period of a periodic point, we will mean the least period of this point).

We consider Sarkovskii's ordering $\to$ on N, defined as follows

$$3 \to 5 \to 7 \to 9 \to \cdots \to 2 \cdot 3 \to 2 \cdot 5 \to 2 \cdot 7 \to 2 \cdot 9 \\ \to \cdots \to 4 \cdot 3 \to 4 \cdot 5 \to 4 \cdot 7 \to \cdots \to \cdots \to 16 \to 8 \to 4 \to 2 \to 1.$$

For every $s \in N$ we denote by S_s the set $\{n \in N\colon s \to n\} \cup \{s\}$. Also we define $S_{2^\infty} = \{1, 2, 4, \ldots, 2^n, \ldots\}$. Similarly, for every $b \in N$ we denote by B_b the set $\{n \in N\colon b \leqslant n\}$, and we write $B_\infty = \varnothing$.

Let $f \in C_1(S^1)$ and let F be a lifting of f. If x is a periodic point of f of period n and X is a real number which satisfies that $\exp(2\pi i X) = x$, then we have $F^n(X) = X + k$ for some $k \in Z$. We shall call the number k/n the rotation number of x and denote it by $\rho(x)$ or $\rho_F(x)$. We denote by $L(f)$ or $L_f(f)$ the set of all rotation numbers of periodic points of f. The following statements are known (see **[BGMY** and **M]**).

(1) $\rho(x)$ does not depend on the choice of X.
(2) If $F' = F + m$, then $\rho_{F'}(x) = \rho_F(x) + m$.
(3) $\rho_{F^m}(x) = m\rho_F(x)$.
(4) If $a < b < c$, $a, c \in L(f)$ and $b \in Q$, then $b \in L(f)$.
(5) $L(f) \cap Z \neq \varnothing$ if and only if $1 \in P(f)$.

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