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FORWARD TRIPLETS AND TOPOLOGICAL ENTROPY ON TREES

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Abstract. We provide a new and very simple criterion of positive topological entropy for tree maps. We prove that a tree map f has positive entropy if and only if some iterate f^k has a periodic orbit with three aligned points consecutive in time, that is, a triplet (a, b, c) such that $f^k(a) = b$, $f^k(b) = c$ and b belongs to the interior of the unique interval connecting a and c (a *forward triplet* of f^k). We also prove a new criterion of entropy zero for simplicial n-periodic patterns P based on the non existence of forward triplets of f^k for any $1 \leq k < n$ inside P. Finally, we study the set $\mathcal{X}_n$ of all n-periodic patterns P that have a forward triplet inside P. For any n, we define a pattern that attains the minimum entropy in $\mathcal{X}_n$ and prove that this entropy is the unique real root in $(1, \infty)$ of the polynomial $x^n - 2x - 1$.

1. **Introduction and statement of the main results.** This paper deals with discrete dynamical systems defined by the iteration of continuous self-maps on trees. We will give some results relating the positive/zero character of the topological entropy of a map to the combinatorial behavior (or *pattern*) of its periodic orbits. In this section we informally introduce some basic notions and present the main results of the paper.

An *interval* is any space homeomorphic to $[0, 1] \subset \mathbb{R}$. A *tree* is a compact uniquely arcwise connected space which is a union of a finite number of intervals.

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