THE BIFURCATIONS OF A PIECEWISE MONOTONE FAMILY OF CIRCLE MAPS RELATED TO THE VAN DER POL EQUATION

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Abstract

By using symbolic dynamics we describe the bifurcations of a family of continuous circle maps. This provides an approximation to the description of the qualitative behavior for a system of the Van der Pol type.

1 Introduction.

We study the bifurcations in a three parameter family $f = f(.,b,\delta)$ of C^0 maps of the circle into itself of degree one, with the parameters ranging in $b_1 \leq b \leq b_2, 0 < \delta \leq \overline{\delta}$, and satisfying the following properties:

There exist $\gamma > 1$, $k > 1/\gamma$, $c > 0$ and an interval $\Delta \subset \mathrm{S}^1$ whose endpoints depend on b and δ such that $|\Delta| < \delta$ and

$$f'(x) > k\gamma \text{ for all } x \in \Delta \tag{1.1}$$

$$-1 + c < f'(x) < -1/\gamma \text{ for all } x \in \mathrm{S}^1 \setminus \Delta \tag{1.2}$$

$$-d/db[f(x_i(b),b,\delta) - x_i(b)] > \omega > 0,\ i = 1,2 \tag{1.3}$$

where $x_1(b)$ and $x_2(b)$, are the endpoints of Δ, all differentiable in b, and $\omega = \omega(\delta)$ is independent of b (see Figure 1.1).

This family is a piecewise-differentiable version of Levi's circle maps (see [L] p.30-31 or [GH] p.74-82) which is used to study the following system of the Van der Pol type with periodic forcing term (see [L]):

$$\epsilon\ddot{x} + \Phi(x)\dot{x} + \epsilon x = bp(t) \tag{1.4}$$

where $\epsilon > 0$ is a small parameter, Φ (damping) is negative for $|x| < 1$ and positive elsewhere, $p(t)$ is periodic of period T and b varies in some finite interval $[b_1, b_2]$. In particular Φ and p can be chosen close (in some sense) to the functions $\Phi_0 = sgn(x^2 - 1)$, $p_0 = sgn\sin(2\pi t/T)$.