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Research paper

An algorithm to compute rotation intervals of circle maps[☆]

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ABSTRACT

In this article we present an efficient algorithm to compute rotation intervals of circle maps of degree one. It is based on the computation of the rotation number of a monotone circle map of degree one with a constant section. The main strength of this algorithm is that it computes *exactly* the rotation interval of a natural subclass of the continuous non-invertible degree one circle maps.

We also compare our algorithm with other existing ones by plotting the Devil's Staircase of a one-parameter family of maps and the Arnold Tongues and rotation intervals of some special non-differentiable families, most of which were out of the reach of the existing algorithms that were centred around differentiable maps.

1. Introduction

The rotation interval plays an important role in combinatorial dynamics. For example Misiurewicz's Theorem [1] links the set of periods of a continuous lifting F of degree one to the set $M := \{n \in \mathbb{N} : \frac{k}{n} \in Rot(F)$ for some integer $k\}$, where $Rot(F)$ denotes the rotation interval of F. Moreover, it is natural to compute lower bounds of the topological entropy depending on the rotation interval [2]. In any case, the knowledge of the rotation interval of circle maps of degree one is of theoretical importance.

The rotation number was introduced by H. PoincarĘ to study the movement of celestial bodies [3], and since then has been found to model a wide variety of physical and sociological processes. In the physical sense, it has been recently applied to climate science [4]. In the sociological one, the application to voting theory [5,6] is specially surprising in this context.

The computation of the rotation number for invertible maps of degree one from $\mathbb{S}^1$ onto itself is well studied, and many very efficient algorithms exist for its computation [7–10]. However, there is a lack of an efficient algorithm for the non-invertible and non-differentiable case.

In this article, we propose a method that allows us to compute the rotation interval for the non-invertible case. Our algorithm is based on the fact that we can compute *exactly* the rotation number of a natural subclass of the class of continuous non-decreasing degree one circle maps that have a constant section and a *rational rotation number*. From this algorithm we

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