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# On the Periodic Orbits of the Perturbed Two- and Three-Body Problems

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**Abstract:** In this work, a perturbed system of the restricted three-body problem is derived when the perturbation forces are conservative alongside the corresponding mean motion of two primaries bodies. Thus, we have proved that the first and second types of periodic orbits of the rotating Kepler problem can persist for all perturbed two-body and circular restricted three-body problems when the perturbation forces are conservative or the perturbed motion has its own extended Jacobian integral.

**Keywords:** restricted 3−body problem; first- (second)-type periodic orbits; conservative forces; first integral of motion

## 1. Introduction

In *celestial mechanics*, the periodic orbits have a considerable importance due to the existence of a direct relationship among the periodic orbits and the motion of planetary systems, as well as the motion of most stellar systems. Thus, in the literature of celestial mechanics, there are many studies on periodic orbits. In 1897, Henri Poincaré studied periodic orbits, and he considered that the exploration of such orbits is important for the understanding of the dynamics of differential systems. Through his study on the restricted three-body problem, he distinguished three classes of periodic orbits [1]:

- First, those emerging from the circular periodic orbits of the rotating Kepler problem;
- Second, those coming from the elliptical periodic orbits of the rotating Kepler problem;
- Third, those generated from the spatial rotating Kepler problem, where the plane of motion is inclined.

The analytical studies on these three types of periodic orbits are developed in [2,3].

In space science, the importance of periodic orbits is not only restricted to *celestial mechanics* but also to *astrodynamics*. There is a need to find periodic orbits to design different types of space missions. For example, in [4], the presence of periodic orbits of the first type is confirmed using the small-parameter technique when the period of a spacecraft's motion and the undisturbed circular orbit of the primaries are equal. Furthermore, the obtained results can be applied to the design and ballistic analysis of electrically powered thrusting spacecraft. Additionally, polar periodic orbits with $\pi/2$ inclination are employed to observe planet surfaces subjected to space missions [5,6].

Periodic solutions or periodic orbits play outstanding roles not only in physical, mathematical, engineering systems but also in biological biosphere systems, where a paramount problem is to estimate or explore if the automatic oscillatory activity can be continued when it is subjected to a small external effect. Many researchers have studied the perseverance and continuation problems of the periodic orbits. Although these studies are

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