ANALYTIC CONTINUATION OF CIRCULAR KEPLER MOTION TO THE GENERAL THREE-DIMENSIONAL THREE-BODY PROBLEM

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ABSTRACT

Families of symmetric periodic solutions of the general three-body problem in three-dimensional space are shown to exist in a rotating frame by analytic continuation of circular Keplerian orbits. The parameter along each family is the relative inclination of the orbital planes.

1. Introduction

The main purpose of this work is to show that there exist families of solutions of the general three-body problem in which two bodies of small mass move in almost circular orbits around a central body of much larger mass, the angle between the orbital planes being arbitrary (see Theorem 2). The method used is the classical analytic continuation applied to the equations which stem from the symmetries of the problem.

This work is a straightforward generalization of a similar one by Jefferys[1] on the three-dimensional restricted three-body problem, in which one of the small bodies is actually of zero mass. As in the restricted case, a rotating frame of reference is used although its rotation is not uniform. The reason is that the z-axis is taken parallel to the angular momentum vector of the system in order to simplify the symmetry conditions (see Theorem 1) and this rules out the possibility of uniform rotation when the elimination of one of the angular variables is performed.

Mention must be made of a result of Poincaré[3] in which a continuity argument (based on a theorem of Kronecker) is used to show the existence of symmetric periodic solutions of the problem. Poincaré defines symmetric conjunction at time t_0 as a position in which the velocities are perpendicular to the plane defined by the three bodies. If at time $t_0 + T$ the system is again in symmetric conjuction, then at time $t_0 + 2T$ the three bodies will be in the same relative position of time T_0, with the same velocities, although the plane