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PHASE PORTRAITS OF PLANAR CONTROL SYSTEMS

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1. INTRODUCTION

Systems of the form

$$\mathbf{x}' = A\mathbf{x} + \varphi(\mathbf{k} \cdot \mathbf{x})\mathbf{b}, \tag{1}$$

where A is a $n \times n$ real matrix and $\mathbf{x}$, $\mathbf{k}$, $\mathbf{b}$ are in $\mathbb{R}^n$ and "$\cdot$" denotes the usual inner product, are of great importance in direct control [1]. The divergence from linearity of these systems results from the presence of the *characteristic function* φ of the control mechanism. The purpose of this mechanism is to improve the asymptotic stability behavior of the equilibrium located at the origin. The reader is referred to Lefschetz [1] and to Narendra and Taylor [2], for details in control theory.

In this paper, we will focus our attention to the class $\mathfrak{FS}$ of two-dimensional systems (1), i.e. $n = 2$, such that

(a) have the origin as an asymptotically stable equilibrium point;
(b) the characteristic function φ has the form

$$\varphi(v) = -u, \quad \text{for } v \le -u; \qquad \varphi(v) = v, \quad \text{for } -u \le v \le u; \qquad \varphi(v) = u, \quad \text{for } u \le v,$$

where u is positive and fixed throughout this paper.

Such systems will be referred to as *fundamental systems* (FS). They are *symmetric* with respect to the origin. That is, their solutions are exchanged under the mapping $\mathbf{x} \to -\mathbf{x}$. Orbits which are invariant under this mapping will be called symmetric.

The characteristic function φ induces a partition of $\mathbb{R}^2$ into three open strips and two straight lines, as follows:

$$\mathbb{S}_- = \{\mathbf{x} : \mathbf{x} \cdot \mathbf{k} < -u\}, \qquad \mathbb{S}_0 = \{\mathbf{x} : -u < \mathbf{x} \cdot \mathbf{k} < u\}, \qquad \mathbb{S}_+ = \{\mathbf{x} : \mathbf{x} \cdot \mathbf{k} > u\}$$

$$\Gamma_- = \{\mathbf{x} : \mathbf{x} \cdot \mathbf{k} = -u\}, \qquad \Gamma_+ = \{\mathbf{x} : \mathbf{x} \cdot \mathbf{k} = u\}.$$

Therefore, the FS splits into the following linear systems:

$$\mathbf{x}' = A\mathbf{x} - u\mathbf{b}, \qquad \text{in } \mathbb{S}_- U\Gamma_- \tag{2.$-$}$$

$$\mathbf{x}' = A\mathbf{x} + (\mathbf{k} \cdot \mathbf{x})\mathbf{b}, \qquad \text{in } \Gamma_- U\mathbb{S}_0 U\Gamma_+ \tag{2.0}$$

$$\mathbf{x}' = A\mathbf{x} + u\mathbf{b}, \qquad \text{in } \mathbb{S}_+ U\Gamma_+. \tag{2.$+$}$$