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An analytic estimation for the instability interval near second order resonances in the restricted three-body problem

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1. Introduction

The planar circular restricted problem of three bodies is defined by a Hamiltonian function which depends on a real parameter μ called the mass ratio, $0 < \mu \le 1/2$. For $\mu = 0$ the problem reduces to the central force problem defined by the Kepler potential in a synodic coordinate system. We shall restrict our attention to direct orbits, that is, orbits whose orientation in an inertial system is the same as the rotation of the synodic one. Direct circular orbits form a family of periodic orbits in the synodical system parameterized by the radius. Elliptic orbits are periodic in the synodic system only if their mean motion n is a rational number $n = p/q$, in which case the orbit is said to be resonant of order $|p - q|$. For each such n they form a family of periodic orbits with constant semiaxis and variable eccentricity.

Periodic orbits of the first kind are obtained by analytic continuation of circular orbits; see [9] and [11] for reference. Those of second kind are obtained from elliptic orbits which cross perpendicularly the synodic x-axis, see [1] and [11]. Both kinds of orbits are symmetric with respect to the x-axis and can be conveniently represented by their characteristic curves in a $(a, \tilde{e})$ plane, where a is the major semiaxis and $\tilde{e}$ is plus or minus the eccentricity according as the pericenter passage occurs at conjunction or opposition in the synodic system (see [8]). In this diagram, for $\mu = 0$, circular orbits are represented by the positive a-semiaxis while elliptic orbits are vertical segments crossing the family of circular orbits at the resonant values of a.

Analytic continuation of circular orbits is not possible at first order resonances: the characteristic curve of first kind periodic orbits breaks at these points and connects with a branch of the second kind family. This result, which was already known from numerical experience, was first derived analytically by Guillaume (see [3] and [10]). Apart from the vicinity of these resonances, where they cease to exist, periodic orbits of the first