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# PERIODIC ORBITS FOR THE PLANAR NEWTONIAN THREE-BODY PROBLEM COMING FROM THE ELLIPTIC RESTRICTED THREE-BODY PROBLEMS

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ABSTRACT. Through the introduction of a new coordinate system followed by a Poincaré compactification, a new relationship is developed to connect the planar three-body problem with the various planar restricted three-body systems. This framework is further used to develop new conditions for the continuation of symmetric periodic orbits from elliptic restricted systems to the full planar three-body problem.

## 1. INTRODUCTION

The purpose of this paper is to study the relationship between the planar periodic orbits of the general Newtonian three-body problem and the planar periodic orbits that arise in different kinds of restricted three-body problems. We make two kinds of contributions. The first is to establish a new and sharper mathematical relationship between the restricted problems and the general three-body problems. We do this by using an appropriate coordinate system so that each of the different kinds of restricted problems (circular, elliptic, etc.) are on the "boundary" of the general three-body problem. Then we use this coordinate system to continue periodic orbits from any elliptic restricted three-body problem to the general three-body problem.

The derivation of $N$-body periodic orbits by continuing periodic orbits (symmetric or nonsymmetric) from the restricted to the general three-body problem is a classical theme that has been studied by several authors. In particular, we call attention to works of Poincaré [Po], Hadjidemetrious [Ha], and Meyer [M1,M2] and the papers they reference. However, for the most part, most of the previous research concerning the continuation of periodic orbits impose strong conditions on the orbit of the restricted problem such as requiring them to be essentially circular or that one particle must be either very close to one of the primaries or very far from both primaries, etc. Our conditions hold for a wide class of orbits; it depends only on a certain property of the initial elliptic restricted three-body problem. To develop these ideas, we analyze the continuation of symmetric periodic orbits (Theorem 6.2). Our periodic orbits

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