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ABELIAN INTEGRALS OF QUADRATIC HAMILTONIAN VECTOR FIELDS WITH AN INVARIANT STRAIGHT LINE*

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Abstract

We prove that the lowest upper bound for the number of isolated zeros of the Abelian integrals associated to quadratic Hamiltonian vector fields having a center and an invariant straight line after quadratic perturbations is one.

1. Introduction

Let $H(x,y)$ be a real polynomial of degree $n+1$, and let $P(x,y)$ and $Q(x,y)$ be real polynomials of degree at most m. The problem of finding an upper bound $N(n,m)$ for the number of isolated zeros of the Abelian integrals

$$I(h) = \int_{\Gamma_h} Q(x,y)\,dx - P(x,y)\,dy, \tag{1.1}$$

where Γ_h varies in the compact components of $H^{-1}(h)$ is called the *weakened 16th Hilbert problem.* It was posed by Arnold in [**1**].

The weakened 16th Hilbert problem is closely related to the problem of determinating an upper bound for the number of limit cycles of the perturbed Hamiltonian system

$$(1.2)_\varepsilon \qquad \begin{aligned} \frac{dx}{dt} &= \frac{\partial H}{\partial y} + \varepsilon P(x,y), \\ \frac{dy}{dt} &= -\frac{\partial H}{\partial x} + \varepsilon Q(x,y), \end{aligned}$$

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