Planar Vector Fields: Hamiltonian Systems, Polynomial Foliations and Structural Stability

Xavier Jarque
Jaume Llibre

October 17th 1995

The first chapter is related with the structurally stable problem for planar Hamiltonian vector fields. In a primer work we consider the polynomial case and characterize the structurally stable systems under $\mathcal{C}^r$ perturbations, polynomial perturbations and Hamiltonian polynomial perturbations. In the later two cases we consider the compactified systems as well as the systems on the open manifold $\mathbb{R}^2$, and consequently we describe the role of the topology. In a second work, we study the $\mathcal{C}^r$ stability of $\mathcal{C}^r$ Hamiltonian vector fields on the plane. In both works we prove that the requirement in the definition of the structural stability that the homeomorphism between the systems and its neighborhoods has to be taken close to the identity is not necessary.

In the second chapter we consider planar polynomial vector fields without finite singular points (they are known as *polynomial foliations*). For this family we upper and lower bound the maximum number of inseparable leaves (that is, orbits in the boundary of Reeb components) as a function of the degree of the polynomial. The general answer to this question is still open. Finally, we also characterize the structurally stable planar polynomial foliations and bound (here we get the sharp bound) the number of inseparable leave on this family. The sharp bound is got by giving an example of an structurally stable planar polynomial foliation with exact n inseparable leaves.

The third and last chapter is dedicate to prove that the sharp bound for the maximum number of inseparable leaves for cubic planar polynomial foliation is 3. Here we use the blow up technique and the classification of cubic systems to reduce the problem. Essentially we draw all possible local phase portrait at the infinite singular points in the Poincaré sphere and show that by taking into account the possible saddle connections the bound is 3. We notice that the main problem relates on the fact that the cases for which the number of inseparable leaves is large coincide with the cases where the infinite singular points are very degenerate and the blow up technique although it is very convenience it requires a lot of work.