POINCARÉ COMPACTIFICATION OF THE KEPLER AND THE COLLINEAR THREE BODY PROBLEMS

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INTRODUCTION

The vector field of the n-body problem can always be written as a polynomial vector field. The idea is to regularize binary collisions in such a way that collisions involving more than two bodies become critical points of the new vector field, [Heg]. For three or more bodies, this procedure introduces redundant variables.

In this paper we describe a method to extend analytically the vector field to a compact manifold. For this purpose, we apply Poincaré compactification of Euclidean phase space into a sphere for the polynomial vector field of the n-body problem.

In Section 2 we study the Kepler problem in the line and in the plane. In both cases we obtain the same results as applying blow up, except that we do not have to fix an energy level.

In Section 3 we study the collinear three body problem. We decompose the compact and connected set of all equilibrium points into five pieces in order to identify and interpret them.

1. POINCARÉ COMPACTIFICATION OF A POLYNOMIAL HAMILTONIAN VECTOR FIELD

Let $X = (P^1, P^2, \ldots, P^n)$ be a polynomial vector field in $\mathbb{R}^n$ and let $m = \max(\deg(P^1), \ldots, \deg(P^n))$ be the degree of X. We denote the time parameter by $\bar{t}$, since more than one time rescaling will be usually involved.

Consider the hyperplane $\Pi = \{y \in \mathbb{R}^{n+1} : y_{n+1} = 1\}$ in $\mathbb{R}^{n+1}$ and let $S^n = \{y \in \mathbb{R}^{n+1} : \|y\| = 1\}$. We take the central projection from the sphere S^n to the hyperplane Π. That is, for each point in Π, we draw the line through this point and the origin in $\mathbb{R}^{n+1}$ and obtain two antipodal points in S^n. Notice that the infinity in the hyperplane corresponds to the equator in the sphere. Of course, if the antipodal points are identified, we obtain the corresponding projective space. For our purposes it is preferable to work with both north and south hemispheres.