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The Global Flow of the Hyperbolic Restricted Three-Body Problem

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Abstract

Two mass points of equal masses $m_1 = m_2 > 0$ move under Newton's law of attraction in a non-collision hyperbolic orbit while their center of mass is at rest. We consider a third mass point, of mass $m_3 = 0$, moving on the straight line L perpendicular to the plane of motion of the first two mass points and passing through their center of mass. Since $m_3 = 0$, the motion of m_1 and m_2 is not affected by the third, and from the symmetry of the motion it is clear that m_3 remains on the line L. The hyperbolic restricted 3-body problem is to describe the motion of m_3. Our main result is the characterization of the global flow of this problem.

1. Introduction

Let $m_1 = m_2 = 1$ be two mass points moving under Newton's law of attraction in hyperbolic orbits in the (x, y)-plane while their center of mass is fixed at the origin of coordinates. As usual these two masses are called *primaries*. We suppose that the hyperbolic orbit is a non-collision orbit; that is, the motion of m_1 and m_2 is not on a straight line, or equivalently, their angular momentum is different from zero. We consider a third mass point with zero mass moving on the z-axis (see Figure 1.1). Since $m_3 = 0$, the motion of the first two mass points is not affected by the third and from the symmetry of the motion it is clear that the third mass point remains on the z-axis. Our problem is to describe the motion of the infinitely small mass. We call this restricted three-body problem the *hyperbolic restricted problem.* The equations of motion of this problem in the phase space $(z, \dot{z}, t)$ are given in Section 2.

We show that the flow of the hyperbolic restricted problem is complete, that is, each solution of the hyperbolic restricted problem is defined for all time t (see Section 3). After some preliminary results in Sections 4 and 5, we prove in Section 6 that when $t \to \pm\infty$, the infinitesimal body either escapes to infinity (hyperbolic motion) or tends to a finite position (parabolic motion). The set of parabolic orbits is a 2-dimensional submanifold of the phase space $(z, \dot{z}, t)$; see Section 7.

We say that an orbit of the infinitesimal mass has type $\mathscr{P}^- n \mathscr{H}^+$ if the orbit is parabolic when $t \to -\infty$, hyperbolic when $t \to +\infty$, and intersects exactly