CHAOS IN THE HYPERBOLIC RESTRICTED 3-BODY PROBLEM

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Let $m_1 = m_2 = 1$ be two mass points moving under Newton's law of attraction in hyperbolic orbits in the xy-plane while their center of mass is fixed at the origin of coordinates. As usual these two masses are called *primaries.* We suppose that the hyperbolic orbit is a non-collision orbit; that is, the motion of m_1 and m_2 is not on a straight line, or equivalently, their angular momentum is different from zero. We consider a third mass point with infinitely small mass moving on the z-axis (see Figure 1). Since $m_3 = 0$ the motion of the first two mass points is not affected by the third and from the symmetry of the motion it is clear that the third mass point will remain on the z-axis. The problem is to describe the motion of the infinitely small mass, and then we have a restricted three-body problem that we call *the hyperbolic restricted problem.* The equation of motion of this problem in the phase space $(z, \dot{z}, t)$ is

$$\ddot{z} = -\frac{16z}{(x(t)^2 + y(t)^2 + 4z^2)^{\frac{3}{2}}}, \tag{1}$$

where $(x(t), y(t))$ is a non-rectilinear hyperbolic orbit solution of the 2-body problem.

This note is a summary of the paper Cors and Llibre[1]. In Cors and Llibre[1] we showed that the flow of the hyperbolic restricted problem is complete, that is each solution of the hyperbolic restricted problem is defined for all time t when $-\infty < t < +\infty$.

From Newton to Chaos, Edited by A.E. Roy
and B.A. Steves, Plenum Press, New York, 1995