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PERIODS AND LEFSCHETZ ZETA FUNCTIONS

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The goal of this paper is to obtain information on the set of periods for a transversal self-map of a compact manifold from the associated Lefschetz zeta function in the case when all its zeros and poles are roots of unity.

1. Introduction and statement of the results. One of the most useful theorems for proving the existence of fixed points or, more generally, periodic points of a transversal self-map f of a compact manifold is the Lefschetz fixed point theorem. When studying the periodic points of f, i.e., the set

$$\mathrm{Per}(f) = \{m \in \mathbb{N} \colon f \text{ has a periodic orbit of minimal period } m\},$$

it is convenient to use the Lefschetz zeta function of f, $Z_f(t)$, which is a generating function for the Lefschetz numbers of all iterates of f. The function $Z_f(t)$ is rational in t and can be computed from the homological invariants of f (see §3).

We shall study C^1 self-maps f of a compact manifold which have only transversal periodic points, so called because the graph of f^m is transverse to the diagonal for all $m > 0$. The main contribution of this paper is the study of the periodic orbits of f when its Lefschetz zeta function has a finite factorization into terms of the form $(1 \pm t^n)^{\pm 1}$. A key point is the introduction of the notion of irreducible factor (see §3 for a precise definition). Our main result is the following.

THEOREM A. *Let $f \colon M \to M$ be a transversal map of a compact manifold. Suppose that all the zeros and poles of its Lefschetz zeta function $Z_f(t)$ are roots of unity, and that $Z_f(t)$ has an irreducible factor of the form $(1 \pm t^n)^{\pm 1}$.*

(a) *If n is odd then $n \in \mathrm{Per}(f)$.*

(b) *If n is even then $\{\frac{n}{2}, n\} \cap \mathrm{Per}(f) \neq \varnothing$.*

The proof of this theorem will be given in §3. From Theorem A it follows that each irreducible factor of the form $(1 \pm t^n)^{\pm 1}$ of the