ON THE FAMILIES OF PERIODIC ORBITS WHICH BIFURCATE FROM THE CIRCULAR SITNIKOV MOTIONS

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Abstract. In this paper we deal with the circular Sitnikov problem as a subsystem of the three–dimensional circular restricted three–body problem. It has a first analytical part where by using elliptic functions we give the analytical expressions for the solutions of the circular Sitnikov problem and for the period function of its family of periodic orbits. We also analyze the qualitative and quantitative behavior of the period function. In the second numerical part, we study the linear stability of the family of periodic orbits of the Sitnikov problem, and of the families of periodic orbits of the three–dimensional circular restricted three–body problem which bifurcate from them; and we follow these bifurcated families until they end in families of periodic orbits of the planar circular restricted three–body problem. We compare our results with the previous ones of other authors on this problem. Finally, the characteristic curves of some bifurcated families obtained for the mass parameter close to $1/2$ are also described.

Key words: Sitnikov motions – periodic orbits – stability – bifurcations

1. Introduction and Statements of the Main Results

We consider the three–dimensional circular restricted three–body problem (RTBP), where the two primaries m_1 and m_2 have masses $1 - \mu$ and μ, respectively. It is assumed that m_1 and m_2 perform uniform circular motion of constant rotational frequency 1. By introducing in R^3 a rotating coordinate system of frequency 1, around the vertical z-axis orthogonal to the (x, y)-plane defined by the motion of the two primaries and passing through the center of mass, we fix the positions of m_1 and m_2 at $(-\mu, 0, 0)$ and $(1 - \mu, 0, 0)$, respectively. The equations of motion for the particle m_3 of zero mass can be written in vectorial notation

$$\mathbf{x} = \mathbf{f}(\mathbf{x}) \tag{1}$$

with $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5, x_6)$, $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = \dot{x}$, $x_5 = \dot{y}$, $x_6 = \dot{z}$, and $\mathbf{f} = (f_1, f_2, f_3, f_4, f_5, f_6)$ where

$$f_1 = x_4, \quad f_2 = x_5, \quad f_3 = x_6,$$

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