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Periods of maps on trees with all branching points fixed

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Abstract. We characterize all possible sets of periods for all continuous self-maps on a tree having all branching points fixed. This result solves a problem which was originally posed by Alsedà, Llibre and Misiurewicz.

1. *Introduction and statement of the results*

In the 1960s Šarkovskiĭ [**Sa**] proved a remarkable theorem about the interrelationships of periodic points of continuous maps on the closed unit interval. Let $\leftarrow$ (the *Šarkovskiĭ ordering*) be the following linear ordering of the positive integers (a more precise definition will be given below):

$$1 \leftarrow 2 \leftarrow 2^2 \leftarrow 2^3 \leftarrow \cdots \leftarrow 7 \cdot 2^2 \leftarrow 5 \cdot 2^2 \leftarrow 3 \cdot 2^2 \leftarrow \cdots$$

$$\leftarrow 7 \cdot 2 \leftarrow 5 \cdot 2 \leftarrow 3 \cdot 2 \leftarrow \cdots \leftarrow 7 \leftarrow 5 \leftarrow 3.$$

Let $f: X \to X$ be a continuous map on the topological space X. A point x of X will be called *periodic with respect to* f (or just *periodic*, if f is obvious from context) if $f^n(x) = x$ for some integer $n > 0$, where f^n is f composed with itself n times. The least n satisfying the above equality is called the *period* of x. The *orbit* of x is the set $\{f^n(x): n \geq 0\}$, where f^0 is the identity map. We denote by $\mathrm{Per}(f)$ the set $\{n: f \text{ has a point of period } n\}$.

SARKOVSKII'S THEOREM. *Let I be the unit interval.*

(1) *For every continuous map $f: I \to I$, if $k \in \mathrm{Per}(f)$ then $m \in \mathrm{Per}(f)$ for every $m \leftarrow k$.*
(2) *Conversely, if S is any initial segment of the Šarkovskiĭ ordering (i.e. a set of positive integers which is closed under $\leftarrow$-predecessors), then there is a continuous map $f: I \to I$ such that* $\mathrm{Per}(f) = S$.

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