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PERIODS FOR TRIANGULAR MAPS

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We study the sets of periods of triangular maps on a cartesian product of arbitrary spaces. As a consequence we extend Kloeden's Theorem (in a 1979 paper) to a class of triangular maps on cartesian products of intervals and circles. We also show that, in some sense, this is the more general situation in which the Sharkovskiĭ ordering gives the periodic structure of triangular maps.

1. INTRODUCTION

In what follows X will denote the following cartesian product of sets: $\prod_{i=1}^{n} X_i$. A map $f : X \to X$ will be called *triangular* if its i-th component function f^i only depends on the first i variables for $i = 1, 2, \ldots, n$; that is, $f^i(x_1, x_2, \ldots, x_n) = f^i(x_1, x_2, \ldots, x_i)$ for all $(x_1, x_2, \ldots, x_n) \in X$ and all $i = 1, 2, \ldots, n$.

A *fixed point* of a map $f : X \to X$ is a point $x \in X$ such that $f(x) = x$. We say that $x \in X$ is an *m-periodic point* of f if x is a fixed point of f^m but is not a fixed point of f^k for any $1 \leqslant k < m$. The set $\{x, f(x), \ldots, f^{m-1}(x)\}$ will be called an *m-periodic orbit* of f. We denote by $\operatorname{Per}(f)$ the set of periods of all periodic points of f.

This paper deals with the problem of determining the possible sets of periods of triangular maps. By using ideas of Kloeden (see [**11**]) we shall show that, for these maps, each periodic orbit can be decomposed into a "product" of periodic orbits (see Proposition 2.2). From this fact we shall obtain a characterisation of the possible sets of periods of triangular maps (see Corollary 2.3). It turns out that this characterisation is rather complicated and difficult to use. However, if we restrict our attention to a particular class of triangular maps, a much nicer characterisation of the set of periods can be obtained. To be more precise we have to introduce some notation.

Assume that X_i is a closed interval I of the real line or a circle S^1 for each $i = 1, 2, \ldots, n$. Clearly, if all the X_i are intervals, then X is an n-dimensional rectangle and if all the X_i are circles then it is the n-dimensional torus. Otherwise, X is an n-dimensional generalised cylinder. In what follows, when we do not need to distinguish

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