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MINIMAL SETS OF PERIODS FOR TORUS MAPS VIA NIELSEN NUMBERS

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The main results in this paper concern the minimal sets of periods possible in a given homotopy class of torus maps. For maps on the 2–torus, we provide a complete description of these minimal sets. A number of results on higher dimensional tori are also proved; including criteria for every map in a given homotopy class to have all periods, or all but finitely many periods.

1. Introduction. In dynamical systems, it is often the case that topological information can be used to study qualitative properties of the system. This article deals with the problem of determining the set of periods (of the periodic orbits) of a mapping given the homotopy class of the mapping. To fix terminology, suppose f is a continuous self–map on the manifold M. *A fixed point of* f is a point x in M such that $f(x) = x$. We will call x a *periodic point of period* n if x is a fixed point of f^n but is not fixed by any f^k, for $1 \leq k < n$.

Denote by $\mathrm{Per}(f)$ the set of natural numbers corresponding to periods of periodic orbits of f.

Even for circle maps $f : \mathbf{S}^1 \to \mathbf{S}^1$ the relation between the degree of f and the set $\mathrm{Per}(f)$ is interesting and nontrivial (see [**5**], [**3**] and, for more details, [**1**]). Let $\mathbb{N}, \mathbb{Z}$, and $\mathbb{R}$ denote, respectively the natural numbers, integers, and reals. Suppose that f is a circle map of degree d. Then we have

(1) For $d = 1$, f may have no periodic points.

(2) For $d \geq 2$ or $d \leq -3$, $\mathrm{Per}(f) = \mathbb{N}$.

(3) For $d = 0$ or $d = -1$, f has a fixed point.

(4) For $d = -2$, $\mathrm{Per}(f) \supset \mathbb{N} \setminus \{2\}$.

One of the objects of this paper is to study the set $\mathrm{Per}(f)$ for continuous self–maps defined on the m–torus $\mathbb{T}^m = \mathbf{S}^1 \times \overset{(m)}{\cdots} \times \mathbf{S}^1$.