Asymptotic expansion of the Dulac map and time for unfoldings of hyperbolic saddles: coefficient properties

D. Marín and J. Villadelprat

Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08193 Cerdanyola del Vallès (Barcelona), Spain
Centre de Recerca Matemàtica, Edifici Cc, Campus de Bellaterra, 08193 Cerdanyola del Vallès (Barcelona), Spain

Departament d'Enginyeria Informàtica i Matemàtiques, ETSE, Universitat Rovira i Virgili, 43007 Tarragona, Spain

December 21, 2020

Abstract. We consider a $\mathscr{C}^\infty$ family of planar vector fields $\{X_{\hat\mu}\}_{\hat\mu\in\hat W}$ having a hyperbolic saddle and we study the Dulac map $D(s;\hat\mu)$ and the Dulac time $T(s;\hat\mu)$ from a transverse section at the stable separatrix to a transverse section at the unstable separatrix, both at arbitrary distance from the saddle. Since the hyperbolicity ratio λ of the saddle plays an important role, we consider it as an independent parameter, so that $\hat\mu=(\lambda,\mu)\in\hat W=(0,+\infty)\times W$, where W is an open subset of $\mathbb{R}^N$. For each $\hat\mu_0\in\hat W$ and $L>0$, the functions $D(s;\hat\mu)$ and $T(s;\hat\mu)$ have an asymptotic expansion at $s=0$ and $\hat\mu\approx\hat\mu_0$ with the remainder being uniformly L-flat with respect to the parameters. The principal part of both asymptotic expansions is given in a monomial scale containing a deformation of the logarithm, the so-called Ecalle-Roussarie compensator. In this paper we are interested in the coefficients of these monomials, which are functions depending on $\hat\mu$ that can be shown to be $\mathscr{C}^\infty$ in their respective domains and "universally" defined, meaning that their existence is stablished before fixing the flatness L and the unfolded parameter $\hat\mu_0$. Each coefficient has its own domain and it is of the form $((0,+\infty)\setminus D)\times W$, where D a discrete set of rational numbers at which a resonance of the hyperbolicity ratio λ occurs. In our main result, Theorem A, we give the explicit expression of some of these coefficients and to this end a fundamental tool is the employment of a sort of incomplete Mellin transform. With regard to these coefficients we also prove that they have poles of order at most two at $D\times W$ and we give the corresponding residue, that plays an important role when compensators appear in the principal part. Furthermore we prove a result, Corollary B, showing that in the analytic setting each coefficient given in Theorem A is meromorphic on $(0,+\infty)\times W$ and has only poles, of order at most two, along $D\times W$.

Contents

1 Introduction and statements of main results 2

2 Proof of Theorem A 9

3 Poles and residues of the coefficients 29

2010 *AMS Subject Classification*: 34C07; 34C20; 34C23.
Key words and phrases: Dulac map, Dulac time, asymptotic expansion, incomplete Mellin transform.
This work has been partially funded by the Ministry of Science, Innovation and Universities of Spain through the grants PGC2018-095998-B-I00 and MTM2017-86795-C3-2-P, the Agency for Management of University and Research Grants of Catalonia through the grants 2017SGR1725 and 2017SGR1617.