THE 16TH HILBERT PROBLEM FOR DISCONTINUOUS PIECEWISE LINEAR HAMILTONIAN SADDLES AND ISOCHRONOUS CENTERS OF DEGREE TWO SEPARATED BY A STRAIGHT LINE

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ABSTRACT. In this paper we deal with discontinuous piecewise differential systems formed by two differential systems separated by a straight line when one of these two differential systems is a linear Hamiltonian saddle and the other is a quadratic isochronous center. It is known that there are four families of quadratic isochronous centers.

We provide upper bounds for the maximum number of limit cycles that these four classes of discontinuous piecewise differential systems can exhibit, so we have solved the extension of the 16th Hilbert problem to such piecewise differential systems. Moreover at least in two of the four classes of these discontinuous piecewise differential systems the obtained upper bound for the maximum number of limit cycles is reached.

1. INTRODUCTION AND MAIN RESULTS

Consider discontinuous piecewise differential systems of the form

$$(\dot{x}, \dot{y}) = \mathbf{F}(x, y) = \begin{cases} \mathbf{F}^-(x, y) = (f^-(x, y), g^-(x, y)) \text{ if } x \leq 0, \\ \mathbf{F}^+(x, y) = (f^+(x, y), g^+(x, y)) \text{ if } x \geq 0, \end{cases} \tag{1}$$

being bivaluated on the separation line $x = 0$. Following [10] a point $(0, y)$ is a *crossing point* if $f^-(0, y)f^+(0, y) > 0$. A *crossing periodic orbit* is a periodic orbit of the discontinuous differential system (1) that has two crossing points and no more, and a *crossing limit cycle* is an isolated periodic orbit in the set of all crossing periodic orbits of system (1). In all the paper we will say limit cycle instead of crossing limit cycle.

Planar continuous piecewise linear differential systems separated by one straight line appear in several non-linear engineering devices or in mathematical biology, see [6, 7, 33, 35, 34] and the references therein. Their maximal number of limit cycles is one, and there are systems with one limit cycle. So the extension of the 16th Hilbert's problem on the maximum number of limit cycles (see for instance [16, 20, 21] for details) for such continuous piecewise differential systems is solved.

For planar discontinuous piecewise linear differential systems separated by one straight line the extension of the 16th Hilbert's problem is an open problem. This problem has been studied by many authors in the past years and there exists a large bibliography trying to determine how many limit cycles can appear in planar systems separated by one straight line, see for instance [1, 2, 3, 4, 9, 11, 12, 13, 14, 15, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 31] and the references therein. It seems that for this class of discontinuous piecewise linear differential systems the upper bound for their maximal number of limit cycles will be three, but this is an open problem.

An *isochronous center* p is a center (that is, a singularity such that all solutions of the differential system except the singularity are periodic in a neighborhood U of it) such that the period of its periodic orbits is constant for all points in the neighborhood U.

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