FINAL EVOLUTIONS FOR LOTKA–VOLTERRA SYSTEMS IN $\mathbb{R}^3$ HAVING A DARBOUX INVARIANT

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Abstract. We classify all the 3–dimensional Lotka–Volterra systems having a Darboux invariant of the form $x^{\lambda_1}y^{\lambda_2}z^{\lambda_3}e^{st}$, where $\lambda_i, s \in \mathbb{R}$, $s(\lambda_1^2+\lambda_2^2+\lambda_3^2) \neq 0$.

The existence of such kind of Darboux invariants in a differential system allow to determine the α–limits and ω–limits of all the orbits of the differential system.

For this class of Lotka–Volterra systems we can describe completely their phase portraits in the Poincaré ball. As an application we illustrate with an example one of these phase portraits.

1. Introduction and statement of the main results

Lotka–Volterra systems in dimension 3 are defined by the system of ordinary differential equations

$$
\begin{aligned}
\dot{x} &= x(a_0+a_1x+a_2y+a_3z) &&= P(x,y,z),\\
\dot{y} &= y(b_0+b_1x+b_2y+b_3z) &&= Q(x,y,z),\\
\dot{z} &= z(c_0+c_1x+c_2y+c_3z) &&= R(x,y,z),
\end{aligned}
\tag{1}
$$

in the space $\mathbb{R}^3$. At the beginning these systems described the growth rate of populations in a community of three interacting species in population dynamics, where $x(t)$, $y(t)$ and $z(t)$ are the population density of the three species at time t, and $a_i, b_i, c_i,\ i=0,1,2,3$ are real constant numbers. For more details on Lotka–Volterra systems see for instance [16, 21, 29, 30].

The dynamics of the Lotka–Volterra systems (1) are far from being understood, although some dynamics for special families of these

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