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PERIODIC ORBITS AND C^1 NON-INTEGRABILITY OF THE $3D$ VAN DER POL-DUFFING OSCILLATOR SYSTEM

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ABSTRACT. In this paper we study the zero-Hopf bifurcation of the 3-dimensional van der Pol-Duffing oscillator system. We get sufficient conditions for the existence of a periodic orbit bifurcating from the zero-Hopf equilibrium localized at the origin of coordinates, using the averaging theory of second order. We prove the non-existence of C^1 first integrals in a neighborhood of this periodic orbit by using the characteristic multipliers of the variational equations of the original system along that periodic orbit.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

In many disciplines complex oscillatory phenomenon has often been observed, for instance in the celebrated Belousov-Zhabotinskii reaction, in gas-phase reactions, and in electrochemical reactions etc. see [3, 5]. Also small oscillations are often observed when changing a control parameter, see [6]. The three-variable version of the van der Pol-Duffing oscillator serves as a typical model of self-excited oscillations in chemistry, physics, electronics, biology and many other disciplines, see [14].

In this paper the following van der Pol-Duffing oscillator system with a cubic non-linearity in 3-dimension is considered

$$\begin{aligned} \dot{x} &= -m(x^3 - \mu x - y), \\ \dot{y} &= x - y - z, \\ \dot{z} &= \beta y, \end{aligned} \tag{1}$$

where $\beta, m \neq 0$ and μ are parameters. This differential system describes the electronic circuit oscillator, see for details [18]. Its dynamical behaviors was discussed in the works, e.g. [1, 6]. In [4] it was proved that this differential system has no analytic first integrals in a neighborhood of the origin of coordinates, that it has no Darboux first integrals. Moreover the authors of [4] claim that the differential system (1) has no C^1 first integrals in a neighborhood of the equilibria of this system under some assumptions, but the result that they use for proving this C^1 non-integrability only works in the neigborhood of periodic orbits instead of a neighborhood of equilibria.

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