ON THE DYNAMICS OF THE ΛCDM MODEL OF THE UNIVERSE

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ABSTRACT. In this paper we prove that the 3-dimensional ΛCDM model of the universe depending on the parameters $a, b \in [1, 4]$ is completely integrable. Moreover, when $a = b$ we describe the dynamics of all orbits in the positive octant of $\mathbb{R}^3$.

1. Introduction and statement of the main result

In this paper we improve the knowledge on the dynamics of the standard cosmological model of the universe, known as the ΛCDM model.

In the paper of Branković and Mijajlović [3] is studied the ΛCDM model under the hypotheses that the matter content of the universe is formed by barotropic perfect fluids without mutual interactions. More precisely, these authors showed that starting with the well known Friedmann equations [5] and doing the following assumptions:

- the average density of the matter fluid and the average density of the radiation fluid are barotropic perfect fluids without mutual interactions,
- the density of the matter content of the universe is the sum of the average densities of the matter and of the radiation,
- the pressure of the matter content of the universe is the sum of the pressures of the matter and of the radiation, and
- the spatial curvature density is less than or equal to zero,

then from equation (33) of [3] the Friedmann equations can be written as

$$\begin{aligned} \dot{x} &= -x\left(-a + \frac{b}{y} - \frac{2}{z}\right) - a, \\ \dot{y} &= -y\left(-b + \frac{a}{x} - \frac{2}{z}\right) - b, \\ \dot{z} &= -z\left(2 + \frac{a}{x} + \frac{b}{y}\right) + 2. \end{aligned} \tag{1}$$

In the differential system (1) the variables x, y and z denote the inverses of the matter density, the radiation density and the cosmological constant density, respectively. Moreover $a = 1 + 3\omega_m$ and $b = 1 + 3\omega_r$ being ω_m and ω_r the state parameters of

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