NEW FAMILIES OF GLOBAL CUBIC CENTERS

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Abstract. An equilibrium point p of a differential system in the plane $\mathbb{R}^2$ is a center if there exists a neighbourhood U of p such that $U \setminus \{p\}$ is filled with periodic orbits. A difficult classical problem in the qualitative theory of differential systems in the plane $\mathbb{R}^2$ is the problem of distinguishing between a focus and a center.

A global center is a center p such that $\mathbb{R}^2 \setminus \{p\}$ is filled with periodic orbits. Another difficult problem in the qualitative theory of differential systems in $\mathbb{R}^2$ is to distinguish inside a family of centers the ones which are global.

Lloyd, Pearson and Romanovsky characterized when the origin of coordinats is a center for the family of cubic polynomial differential systems

$$\begin{aligned} \dot{x} &= y - Cx^2 + (B+2D)\,xy + Cy^2 + Px^3 + Gx^2y - (H+3P)\,xy^2 + Ky^3, \\ \dot{y} &= -x + Dx^2 + (E+2C)\,xy - Dy^2 - Kx^3 - (H+3P)\,x^2y - Gxy^2 + Py^3. \end{aligned}$$

Here we characterize when the origin of this family of differential system, is a global center.

1. Introduction and Statement of the Main Results

The notion of center first appears in the work of Huygens in 1656 on the pendulum clock (look at [12, 17]), but only with the works of Poincaré (see [18]) in 1881 and Dulac (see [8]) in 1908 the notion of center was rigorously defined.

A polynomial differential system in the plane $\mathbb{R}^2$ of degree n is a differential system of the form

$$\dot{x} = P\,(x,y)\,, \qquad \dot{y} = Q\,(x,y)\,, \tag{1}$$

being P and Q polynomials in the variables x and y. The n is the maximum of the degrees of the polynomials P and Q. As usual the dot denotes derivative with respect to the time t.

[1]2020 *Mathematics Subject Classification.* 34C29, 34C25, 47H11.
Key words and phrases. center, global center, cubic polynomial differential systems.
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