PERIODIC ORBITS OF CONTINUOUS-DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS WITH FOUR PIECES SEPARATED BY THE CURVE $xy = 0$ AND FORMED BY LINEAR HAMILTONIAN SYSTEMS

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ABSTRACT. In recent years there has been a significant interest in studying the piecewise differential systems, mainly due to their wide range of applications in modeling natural phenomena. To understand the dynamics of such systems in the plane is a significant challenge, particularly when we want to study their periodic orbits and, more specifically, their limit cycles. Consequently numerous studies have been dedicated to investigate the existence or non-existence of periodic orbits within continuous and discontinuous piecewise differential systems. However to the best of our knowledge, this paper is one of the pioneering works analysing the periodic orbits within a specific class of piecewise differential systems, the ones exhibiting continuity in one part of the separation line while being discontinuous in the other part.

Our study analyzes the periodic orbits of the piecewise differential systems formed by four pieces, having the curve $xy = 0$ as the separation line, and in each piece there is an arbitrary linear Hamiltonian system. Moreover we assume that these piecewise differential systems exhibit continuity along the x-axis while being discontinuous along the y-axis.

1. Introduction and statement of the main results

A dynamical system is any system that changes over time, and ODEs provide a concise and elegant way to capture this behavior. A dynamical system can be defined a function that describes the time dependence of a point in an ambient space, such as a parametric curve. Examples of dynamical systems that can be modeled with ODEs include the oscillation of a clock pendulum, the flow of water through a pipe, the motion of particles in the air, and the population dynamics of a lake's fish species. By modeling these systems with ODEs, we gain insight into their behavior, which can inform important decisions in fields such as physics, engineering, and ecology. Therefore, ODEs play a crucial role in the study of dynamical systems and are essential tools for understanding the behavior of natural phenomena.

Piecewise differential systems in the plane are a particular type of dynamical system that are defined by different sets of differential equations in different regions of the state space. The boundary between regions, called the switching or separation line defines the conditions under which the system switches from one set of equations to another, see for more details [8, 15, 26]. These systems are often used to model complex phenomena that exhibit different behaviors or dynamics under different conditions, such as ecological systems or mechanical systems subject to switching or control inputs.

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