# GLOBAL CENTERS OF A CLASS OF CUBIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. A difficult classical problem in the qualitative theory of differential systems in the plane $\mathbb{R}^2$ is the center-focus problem, i.e. to distinguish between a focus and a center. Another difficult problem is to distinguish inside a family of centers the ones which are global. A global center is a center $p$ such that $\mathbb{R}^2 \setminus \{p\}$ is filled with periodic orbits.

In this paper we classify the global centers of the family of real polynomial differential systems of degree 3 that in complex notation write

$$i\dot{w} = w - A_3\overline{w}^2 - A_4w^3 - A_5w^2\overline{w} - A_6w\overline{w}^2,$$

where $w = x + iy$ and $A_k \in \mathbb{C}$ for $k = 3, 4, 5, 6$.

## 1. Introduction

The general cubic differential equations in complex notation having either a weak focus or a center at the origin of coordinates are

$$i\dot{w} = w - A_1w^2 - A_2w\overline{w} - A_3\overline{w}^2 - A_4w^3 - A_5w^2\overline{w} - A_6w\overline{w}^2 - A_7\overline{w}^3, \tag{1}$$

where $w = x + iy$ and $A_k \in \mathbb{C}$, $k = 1, \cdots, 7$. These differential systems have been considered in several articles, thus in the papers [2, 12, 13, 15] the authors provide for some subclasses of these differential systems necessary and sufficient conditions in order that the equilibrium point at the origin of coordinates be a center. In this paper we restrict our attention to the subclass $A_1 = A_2 = A_7 = 0$. Then the complex differential equation (1) can be written as the following real cubic polynomial differential system

$$\begin{aligned} \dot{x} = &\ y + 2a_1xy - a_2(x^2 - y^2) - (b_2 + c_2 + d_2)x^3 - (3b_1 + c_1 - d_1)x^2y \\ &+ (3b_2 - c_2 - d_2)xy^2, \\ \dot{y} = &\ -x + a_1(x^2 - y^2) + 2a_2xy + (b_1 + c_1 + d_1)x^3 - (3b_2 + c_2 - d_2)x^2y \\ &+ (-3b_1 + c_1 + d_1)xy^2 + (b_2 - c_2 + d_2)y^3, \end{aligned} \tag{2}$$

in the plane $\mathbb{R}^2$, where $A_3 = a_1 + ia_2$, $A_4 = b_1 + ib_2$, $A_5 = c_1 + ic_2$ and $A_6 = d_1 + id_2$.

The main goal of this paper is to classify the global centers of system (2). We recall that a *center* of a differential equation in $\mathbb{R}^2$ is an equilibrium point $p$ having a neighbourhood $\mathcal{U}$ such that $\mathcal{U} \setminus \{p\}$ is filled with periodic orbits. In particular, a *global center* is a center $p$ such that $\mathbb{R}^2 \setminus \{p\}$ is filled with periodic orbits.

The classical problem of distinguishing between a focus and a center is a difficult one and it is one of the challenges of the theory of nonlinear differential systems in the plane $\mathbb{R}^2$. The rigorous notion of center appeared in the literature with the works of Poincaré [14] and Dulac [5].

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