LIMIT CYCLES BIFURCATING FROM KOLMOGOROV SYSTEMS OF DEGREE 4 IN $\mathbb{R}^2$ AND $\mathbb{R}^3$

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ABSTRACT. We consider the Kolmogorov system of degree 4 in $\mathbb{R}^2$ having an equilibrium point in the positive quadrant and the Kolmogorov system of degree 4 in $\mathbb{R}^3$ having an equilibrium point in the positive octant, note that the equilibrium point is located where the Kolmogorov systems have a biological meaning. We provide the conditions under which the equilibrium point of the system in $\mathbb{R}^2$ will be a Hopf point and the equilibrium point of the system in $\mathbb{R}^3$ and will be a zero-Hopf equilibrium point. Using the averaging theory of first and second order we provide the explicit conditions for the existence of limit cycles that bifurcate from these equilibria and we also characterize the type of stability or instability of these limit cycles.

1. Introduction and statement of the main results

Kolmogorov systems are an extension given in 1936 by Andrey Kolmogorov [10] of Lotka–Volterra systems which have the form $dx_i/dt = x_i(a_{1i}x_1 + a_{2i}x_2 + a_{0i})$ with $i = 1, 2$, and was initially proposed as a model for the study of interactions between two species, independently developed by Alfred J. Lotka in 1925 [14] and Vito Volterra in 1926 [23]. A. Kolmogorov extended Lotka-Volterra systems to arbitrary dimension and degree.

Many applications of Lotka–Volterra and Kolmogorov systems can be cited, their areas of application more know are in biology were have been applied to model diverse natural phenomena such as the evolution between three species in competition (studied by May and Leonard [16]), the time evolution of conflicting species (see for more details May [17] and [9, 24] as examples), also in economics (where [5], [7] and [22] are some examples), and in other areas as the evolution of electrons, ions and neutral species in plasma physics [15], chemical reactions [8], hydrodynamics [4], to name a few.

A *limit cycle* of a differential system is a periodic orbit isolated in the set of all periodic orbits that these differential system could have. A limit cycle can be *stable* or *unstable*, and this depend on the characteristic exponent of the associated Poincaré map, if at least one characteristic exponent has positive real part then the limit cycle is *unstable*, and if all the characteristic exponents has negative real part the limit cycle is *stable*.

The existence of a limit cycle is an important phenomenon in the dynamics of the differential systems, its analysis is significant in the study of the nonlinear differential systems (see for instance [18]), and the Hopf bifurcations are also relevant for finding limit cycles (see for example [20]).

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