DOUBLE ZERO-HOPF BIFURCATION IN A FOUR-DIMENSIONAL HYPERCHAOTIC SYSTEM

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Abstract. The subject of this paper concerns with the bifurcation of limit cycles for a four dimensional hyperchaotic system. This hyperchaotic system depens on five parameters and we restrict our study to the set of parameters where a double zero-Hopf bifurcation may occurs, that is where an isolated equilibrum point has a double zero and a pair of purely imaginary eigenvalues. For doing this study some adequate changes of parameters and coordinates must be done in order that the computations become easier. The main result is based on the averaging theory.

1. Introduction

In 1979, hyperchaos was first presented by Rössler in the hyperchaotic Rössler system [12]. These differential systems are characterized as chaotic systems with more than one positive Lyapunov exponent. This implies that the dynamics of those systems are expanded in more than one direction simultaneously. As a consequence those systems have a more complex chaotic attractor comparing with the chaotic system with only one positive Lyapunov exponent. This expansion of the dynamics, happening at the same time in more than one direction, makes hyperchaotic systems have better performance in many chaos based fields, including applications in technology field, when compared to chaotic systems. For example, hyperchaotic systems, due to higher unpredictability and the structure of the attractors (more complicated than for chaotic systems), can be used to improve the security in chaotic communication systems. In this context the chaotic signal is used to disguise the message to be transmitted, once using chaotic systems this disguise is not always secure [10].

In this paper we study periodic orbits bifurcating from four-dimensional *double zero-Hopf equilibria* (that is, isolated equilibria with a double zero and a pair of purely imaginary eigenvalues) for a hyperchaotic system in $\mathbb{R}^4$.

The zero-Hopf bifurcations have been extensively studied in three-dimensional chaotic systems (see, e.g., [7], and [4] and references therein) and with less emphasis in four-dimensional chaotic systems (in particular, hyperchaotic systems). Some recent works investigating Hopf and zero-Hopf bifurcations of hyperchaotic systems can be found using normal form theory (see, e.g. [5]), Lyapunov exponents (see, e.g. [16]) and averaging theory (see, e.g. [9]).

The hyperchaotic system here studied is the differential system

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