ON THE INDEX OF AN ISOLATED EQUILIBRIUM POINT FOR PIECEWISE POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract. The index of an equilibrium point is an integer, which is a quantity that characterizes the topological structure of the equilibrium point.

There are mainly two methods to calculate the index of an isolated equilibrium point for smooth differential systems. On one hand, Poincaré and Bendixson deduced that the index of an equilibrium point can be obtained from the number of hyperbolic and elliptic sectors in a neighborhood of the equilibrium point, which is known as the Poincaré-Bendixson formula. On the other hand, several works contributed to compute the index using the algebraic method called Cauchy's index.

In this paper we generalize the Cauchy's index to calculate the index of equilibrium point for piecewise polynomial differential systems. The application of these results to piecewise smooth quadratic quasi-homogeneous differential systems is also given.

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1. Introduction and statement of the main results

Consider the following differential system

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = P(x,y), \quad \frac{\mathrm{d}y}{\mathrm{d}\tau} = Q(x,y), \tag{1}$$

where $P(x,y)$ and $Q(x,y)$ are $C^r, r \geqslant 1$ large enough for our purpose. Assume that $O(0,0)$ is an isolated equilibrium point of system (1) (that is, $P(0,0) = Q(0,0) = 0$), we can choose a suitable closed curve σ, such that in the closed region bounded by σ does not contain any equilibrium point other than $O(0,0)$. The index of the equilibrium point $O(0,0)$ is a quantity that provides information on the topological structure of the equilibrium point, which can be defined as follows:

$$I_O := I(O,\sigma) = \frac{1}{2\pi} \oint_\sigma \mathrm{d} \arctan \frac{Q(x,y)}{P(x,y)}. \tag{2}$$

It is worth to note that the index of an equilibrium point I_O is an integer which is independent from the chosen of σ. Knowing the indices of the equilibrium points, we can deduce several important results on the existence and location of limit cycles [18]. For example, assume that system (1) has only isolated equilibrium points, then the sum of the indices of the equilibrium points enclosed by any limit cycle is 1, thus the interior of any limit cycle must contain equilibrium points.