# LIMIT CYCLES FOR CONTINUOUS AND DISCONTINUOUS PERTURBATIONS OF TWO CUBIC ISOCHRONOUS CENTERS

JOHANA JIMENEZ[1] AND JAUME LLIBRE[2]

Abstract. In this paper we consider two reversible cubic isochronous centers and we study both continuous and discontinuous perturbations inside the general class of cubic polnomial differential systems. In the continuous case we provided the maximum number of small and medium limit cycles which bifurcate from the center and from the periodic orbits surrounding the center, respectively, either when both of reversible cubic isochronous centers are perturbed inside the class of all continuous cubic polynomial differential systems or when the parameters of one of the reversible cubic centers are perturbed. In the discontinuous case we study the small limit cycles for six families of discontinuous piecewise differential systems formed from the two reversible cubic isochronous centers. The main tool used for proving our results is the averaging theory up to seven order.

## 1. Introduction and statement of the main results

A *limit cycle* of a differential system is a periodic orbit that is isolated in the set of all periodic orbits of the system. One of the main open problems in the qualitative theory of planar differential systems is the study of the limit cycles that can bifurcate from a center, or from its periodic orbits. These problems have been studied intensively in these last decades and are closely related to the Hilbert's 16th problem, see [6].

A classical and useful tool for studying the behavior of nonlinear smooth differential systems and more precisely their periodic solutions is the averaging theory, see for instance [16, 18]. The first classical results of averaging theory for studying the periodic orbits of differential systems need at least that the systems be of class $C^2$, and moreover, initially the theory was developed up to the second order. In [2] the authors extended the averaging theory for studying orbits to continuous differential systems using the Brouwer degree up to third order, and in the papers [5, 11, 12] the averaging theory for computing periodic orbits is extended to any order.

These last years great interest has appeared in the mathematical community for studying the discontinuous piecewise differential systems and for understanding the rich dynamics of these piecewise differential systems because they are widely used to model processes appearing in real phenomena as in electronics, mechanics, economy, etc., see for instance the books of di Bernardo et al. [4], Simpson [17], the survey of Makarenkov and Lamb [14] and the hundreds of references quoted there.

In order to study the periodic orbits of the discontinuous piecewise differential systems the classical averaging theory has been extended to such differential systems, see for instance [8, 10, 13].

The authors of [7] used the averaging theory up to the six order for studying the limit cycles bifurcating from uniform isochronous cubic centers when they are perturbed either inside the class of all continuous cubic polynomial differential systems, or inside the class of all discontinuous differential systems formed by two cubic differential systems separated

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