PHASE PORTRAITS OF A FAMILY OF HAMILTONIAN CUBIC SYSTEMS

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Abstract. We deal with the one-dimensional parameter family of Hamiltonian cubic polynomial differential systems

$$\dot{x} = y - y(y^2 + 3x^2\mu), \qquad \dot{y} = x + x(x^2 + 3y^2\mu),$$

where $(x, y) \in \mathbb{R}^2$ are the variables and μ is a real parameter. We classify in the Poincaré disc the topological phase portraits of this family of systems when the parameter μ varies, describing the bifurcations which take place.

1. Introduction

A *cubic polynomial differential system* is a system of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

where P and Q are polynomials in the variables x and y, and the maximum of the degrees of P and Q is three.

The phase portraits of the polynomial differential systems (1) of degree 1, i.e. the *linear differential systems*, are well known. There are more than one thousands papers published on the polynomial differential systems (1) of degree 2, i.e. the so called *quadratic systems*, see for instance the books [2, 16, 19] and the hundreds of references quoted in each of these books. With respect to the polynomial differential systems (1) of degree 3, or simply *cubic systems*, few papers have been published if we compare with the papers dedicated to the quadratic systems, but this is changing see for instance some of the papers dedicated to the cubic systems published in 2020 [3, 4, 6, 7, 8, 9, 10, 11, 12, 17, 18, 20, 21] and their references.

Very few papers are dedicated to classify the phase portraits of some families of cubic systems. In this article we classify all topological

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