ZERO–HOPF BIFURCATION IN THE CHUA'S CIRCUIT

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Abstract. An equilibrium point of a differential system in $\mathbb{R}^3$ such that the eigenvalues of the Jacobian matrix of the system at the equilibrium are 0 and $\pm\omega i$ with $\omega > 0$ is called a zero-Hopf equilibrium point.

First we prove that the Chua's circuit can have three zero-Hopf equilibria varying its three parameters. After we show that from the zero-Hopf equilibrium point localized at the origin of coordinates can bifurcate one periodic orbit. Moreover, we provide an analytic estimation of the expression of this periodic orbit and we have determined the kind of the stability of the periodic orbit in function of the parameters of the perturbation. The tool used for proving these results has been the averaging theory of second order.

Unfortunately the averaging theory does not provide information about the possible periodic orbits bifurcating from the other two zero-Hopf equilibria that the Chua's circuit can have.

1. Introduction and statement of the main results

In 1993 Chua *et al.* [7] proposed to replace the piecewise linear function of his famous circuit by a cubic one. The system they designed became then a relaxation oscillator with a cubic nonlinear characteristic elaborated from a circuit comprising a harmonic oscillator for which the operation was based on a field-effect transistor, coupled to a relaxation-oscillator composed of a tunnel diode. The modeling of the circuit used a capacity preventing from abrupt voltage drops and allowing to describe the fast motion of this oscillator. This gave rise to the following equations (1) which constitute a singularly perturbed system and are since considered as the paradigm of complex dynamics [11].

The Chua's circuit is analyzed using the Kirchhoff's laws. Then the dynamics of the Chua's circuit is modeled by means of the following system of three nonlinear ordinary differential equations in the variables $x(t)$, $y(t)$ and $z(t)$:

$$\begin{aligned} \dot{x} &= a(y - cx - x^3), \\ \dot{y} &= x - y + z, \\ \dot{z} &= -by, \end{aligned} \tag{1}$$

where a, b and c are real parameters and the dot indicates derivative with respect to the time t. For more details on the Chua's circuit see the references [3, 4].

Note that this differential system is invariant under the symmetry $(x, y, z) \to (-x, -y, -z)$. Therefore if $(x(t), y(t), z(t))$ is a solution of the differential system

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