ON THE NUMBER OF LIMIT CYCLES FOR PIECEWISE POLYNOMIAL HOLOMORPHIC SYSTEMS

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Abstract. In this paper we are concerned with determining lower bounds of the number of limit cycles for piecewise polynomial holomorphic systems with a straight line of discontinuity. We approach this problem with different points of view: study of the number of zeros of the first and second order averaging functions, or with the control of the limit cycles appearing from a monodromic equilibrium point via a degenerated Andronov–Hoph type bifurcation, adding at the very end the sliding effects. We also use the Poincaré–Miranda theorem for obtaining an explicit piecewise linear holomorphic systems with 3 limit cycles, result that improves the known examples in the literature that had a single limit cycle.

1. Introduction

The models in nonsmooth dynamics of differential equations have attracted the attention of many researchers for the accuracy of the obtained results comparing with the real observations, see more details in the three books [1, 5, 18] and their references. Several of these models are given by piecewise smooth systems with some switching manifold. Moreover, on many of them the smooth systems are linear and the switching manifold is a straight line.

Holomorphic functions have a wide range of applications in several areas of applied science such as the study of fluid dynamics, for more information see, for instance, [3, 6, 9]. One of the most remarkable dynamical properties of holomorphic systems $\dot{z} = f(z)$ is the fact that these systems do not have limit cycles. The study and the properties of these systems make them interesting and beautiful but precisely this absence of limit cycles makes them dynamically poor. However, in [16] the authors proved that there are piecewise linear holomorphic systems that have one limit cycle. Moreover they proved that if the equilibrium points are on the straight line of discontinuity this limit cycle is unique.

In this paper we are interested on piecewise polynomial holomorphic systems (PWHS). On one hand, each of the smooth systems has the beautiful properties of the holomorphic systems but on the other hand considered as a piecewise system they exhibits all the interesting features of the piecewise linear systems and much more. In particular, as we have already explained, they can have limit cycles. Moreover, all the power of complex notation and of complex analysis can be used in their study.

Essentially, the techniques we employed to prove the results of this paper could be applied to piecewise smooth vector fields, without necessarily being piecewise holomorphic. However, the fact that it is holomorphic endows the vector field with important properties that simplify calculations. In fact, holomorphic systems, apart of the absence of limit cycles, have other surprising and interesting properties: reversibility, integrability, all their centers are isochronous, and complete knowledge of the phase portraits around their non-essential singularities, with

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