

THE UNIFORM ISOCHRONOUS CENTERS WITH HOMOGENEOUS NONLINEARITIES OF DEGREE 5

GUANGFENG DONG¹ AND JAUME LLIBRE²

¹ *Department of Mathematics, Jinan University, Guangzhou, 510632, China*

² *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra,
Barcelona, Catalonia, Spain*

ABSTRACT. In this paper we classify the topological phase portraits of polynomial differential systems with a uniform isochronous center, whose nonlinear part is a homogenous polynomial of degree 5. We prove that there are three distinct topological phase portraits in the Poincaré disc for such differential systems.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

In the XVII century people started the interest in the isochronous centers with the works of C. Huygens, see [9]. In fact in many physical problems appears the isochronicity phenomenon, see for instance [5]. A point $p \in \mathbb{R}^2$ is a *center* if it is an equilibrium point of a planar differential system having a neighborhood U of p such that all the orbits in $U \setminus \{p\}$ are periodic. For each point $q \in U \setminus \{p\}$ we denote by $T(q)$ the period of the periodic orbit passing through q . If $T(q)$ is constant for all $q \in U \setminus \{p\}$, then p is called an *isochronous center*. Of course, if p is isochronous

E-mail address: donggf@jnu.edu.cn, jaume.llibre@uab.cat.

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