Invariant manifolds of maps and vector fields with nilpotent parabolic tori

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Abstract

We consider analytic maps and vector fields defined in $\mathbb{R}^2 \times \mathbb{T}^d$, having a d-dimensional invariant torus $\mathcal{T}$. The map (resp. vector field) restricted to $\mathcal{T}$ defines a rotation of frequency ω, and its derivative restricted to transversal directions to $\mathcal{T}$ does not diagonalize. In this context, we give conditions on the coefficients of the nonlinear terms of the map (resp. vector field) under which $\mathcal{T}$ possesses stable and unstable invariant manifolds, and we show that such invariant manifolds are analyitic away from the invariant torus. We also provide effective algorithms to compute approximations of parameterizations of the invariant manifolds, and present some applications of the results.

2020 Mathematics Subject Classification: Primary: 37D10; Secondary: 37C25.
Key words and phrases: Parabolic torus, invariant manifold, parameterization method.

1 Introduction

The study of parabolic invariant manifolds is relevant, apart from the interest that presents itself as a mathematical problem, because this kind of manifolds appears naturally in many problems motivated by physics, chemistry and other sciences.

Parabolic manifolds have been used to prove the existence of oscillatory motions in some well-known problems of Celestial Mechanics as the Sitnikov problem [17, 15] and the circular planar restricted three-body problem [11, 12, 14] using the transversal intersection of invariant manifolds of parabolic points and symbolic dynamics.

The existence of oscillatory motions in all these instances is strongly related to some invariant objects at infinity that are either fixed points, periodic orbits or invariant tori and to their stable and unstable manifolds. Although these invariant objects are parabolic in the sense that the linearization of the vector field on them has all the eigenvalues equal to zero, they do have stable and unstable