GLOBAL NILPOTENT REVERSIBLE CENTERS WITH CUBIC NONLINEARITIES SYMMETRIC WITH RESPECT TO THE y-AXIS

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Abstract. We classify when the polynomial differential systems of the form $\dot{x} = y + P_3(x, y)$, $\dot{y} = Q_3(x, y)$, that are invariant under the change $(x, y, t) \mapsto (-x, y, -t)$ (here P_3 and Q_3 are general polynomials of degree three) have global centers.

1. Introduction

A global center of a polynomial differential system in the plane is a singular point p having $\mathbb{R}^2 \setminus \{p\}$ filled up with periodic orbits. The notion of global center is related with the so-called Jacobian conjecture, see [11]. It is known (see [6, 9]) that global centers only appear in polynomials with odd degree and that the classification of global centers of polynomial vector fields in the plane is a very difficult problem.

The first non-trivial polynomial differential systems of odd degree are the polynomial differential systems of degree three and for them we would like to classify when they have a nilpotent center at the origin. These systems can be written as

$$\begin{aligned} x' &= y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3, \\ y' &= b_{20}x^2 + b_{11}xy + b_{02}y^2 + b_{30}x^3 + b_{21}x^2y + b_{12}xy^2 + b_{03}y^3. \end{aligned} \tag{1}$$

Until now for polynomial differential systems of degree higher than two the complete characterization of centers is not possible and there are only partial results. This enables the possibility to work with the whole systems (1) to classify their global centers. So, up to now only the classification of global centers has been done for the subclass of systems (1) that are either Hamiltonian and symmetric with respect to the y-axis (which forces that a nilpotent center is indeed a center), see [2], or that do not have terms of degree two (for which the classification of centers is known), see [7]. In this paper we go a step further and we provide the complete classification of global nilpotent centers of systems (1) that are invariant under the symmetry $(x, y, t) \mapsto (-x, y, -t)$. For these systems we already know that if the origin is a nilpotent center, then it is a center. There are other papers related with the existence of global centers (see [3, 4, 8] to cite just a few).

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