ON THE LIMIT CYCLES OF 3-DIMENSIONAL PIECEWISE LINEAR DIFFERENTIAL SYSTEMS

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ABSTRACT. We deal with a three–parameter family of symmetric piecewise linear differential systems with respect to the origin of $\mathbb{R}^3$ which appears in control theory. For this family we conjecture the existence and uniqueness of a symmetric limit cycle.

1. Introduction and statement of the main results

A class of differential systems which are relevant in control theory are the Lur'e systems which are symmetric piecewise linear differential system of the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}\varphi(\mathbf{c}^T\mathbf{x}(t)), \tag{1}$$

here $\mathbf{A}$ is a $n \times n$ constant matrix, $\mathbf{b}$ and $\mathbf{c}$ are given vectors in $\mathbb{R}^n$, and the *input* function $\varphi(\mathbf{c}^T\mathbf{x}(t))$ is the feedback of the *output* $\mathbf{c}^T\mathbf{x}(t)$ through the nonlinear continuous function $\varphi : \mathbb{R} \to \mathbb{R}$ defined as

$$\varphi(\sigma) = \sigma \text{ for } |\sigma| \le 1, \qquad \varphi(\sigma) = \operatorname{sgn}(\sigma) \text{ for } |\sigma| > 1. \tag{2}$$

For additional information on the Lur'e systems see for instance [1, 3, 5, 7, 9].

We shall restrict systems (1) to $\mathbb{R}^3$, so that $\mathbf{x}(t) = (x(t), y(t), z(t)) \in \mathbb{R}^3$, and without loss of generality we assume that $\mathbf{c} = (1, 0, 0)^T$.

Due to the definition of the function φ the space $\mathbb{R}^3$ is divided into three zones L, C and R separated by the two planes P_- and P_+, where

$$\begin{aligned} L &= \{(x, y, z) \in \mathbb{R}^3 : x < -1\},\\ P_- &= \{(x, y, z) \in \mathbb{R}^3 : x = -1\},\\ C &= \{(x, y, z) \in \mathbb{R}^3 : -1 < x < 1\},\\ P_+ &= \{(x, y, z) \in \mathbb{R}^3 : x = 1\},\\ R &= \{(x, y, z) \in \mathbb{R}^3 : x > 1\}. \end{aligned}$$

So the differential system (1) is a symmetric piecewise linear differential system formed by the following three pieces

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} - \mathbf{b} \text{ in } L \cup P_-,\\ \dot{\mathbf{x}} &= \mathbf{B}\mathbf{x} \quad\;\;\;\;\, \text{in } P_- \cup C \cup P_+,\\ \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b} \text{ in } P_+ \cup R, \end{aligned} \tag{3}$$

where $\mathbf{B} = \mathbf{A} + \mathbf{b}\mathbf{c}^T$. Since $\varphi(0) = 0$ the origin of coordinates is an equilibrium point of system (1).

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