The transition between two ecological limit cycles in one predator-two competitive prey model

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Abstract

In this paper we analyze an ecological model with a single predator and two competitive prey species. This model incorporates several key elements, including logistic growth dynamics for the prey populations, a Holling type II functional response governing predator-prey interactions, and the inclusion of intraspecific competition among predators.

Our study establishes stringent conditions on the model parameters to ensure the existence of two coexistence equilibrium points (CEPs). Of particular interest is one CEP that undergoes a Hopf bifurcation, resulting in a continuous transition between two limit cycles residing in different dimensions. More precisely, we observe a periodic solution within a three-dimensional phase space, alongside another periodic solution confined to an invariant phase plane. The bialternate sum matrix criterion serves as a vital tool in demonstrating the existence of this Hopf bifurcation.

Furthermore, employing Lyapunov exponents, we provide numerical evidence showcasing the emergence of chaotic dynamics within the model. This comprehensive analysis sheds light on the intricate behavior of the ecological system under consideration, offering valuable insights into the complex interplay of ecological factors and nonlinear phenomena within the predator-prey dynamics that has not been previously detected.

***Keywords*:** Ecological model, limit cycle, biproduct, Hopf bifurcation, stability.

1. Introduction

The dynamics of the trophic relationships have been widely studied during these last years, with predator–prey interactions standing out as recurrent subjects due to their biological significance (see Refs. [1–7]). Another important type of interaction is competition, which can manifest as interspecific or intraspecific. Interspecific mode of this interaction, arises when individuals within a community vie for the same limited resources in a particular habitat, while intraspecific involves competition among members of the same population (see Refs. [8, 9]).

Understanding the local dynamics of these population phenomena hinges on the underlying assumptions employed in their modeling, including factors as mortality rates, environmental conditions, the type of functional response, and numerous other variables. For instance, adopting a specific functional response in our model (see Refs. [10–14]) carries ecological

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