THE MATCHING OF TWO MARKUS-YAMABE PIECEWISE SMOOTH SYSTEMS IN THE PLANE

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Abstract. A Markus-Yamabe vector field is a smooth vector field in $\mathbb{R}^n$ having only one equilibrium point and such that the spectrum of its Jacobian matrix at any point of $\mathbb{R}^n$ is on the left of the imaginary axis in the complex plane. A vector field is globally asymptotically stable if it has an equilibrium point p and all the other orbits tend to p in forward time. One of the great results of the Qualitative Theory of Differential Equations establishes that a planar Markus-Yamabe vector field is globally asymptotically stable, but a Markus-Yamabe vector field defined in $\mathbb{R}^n$, $n \geqslant 3$, does not have in general this property. We prove that planar crossing piecewise smooth vector fields defined in two zones formed by two Markus-Yamabe vector fields sharing the same equilibrium point located on the separation straight line are not necessarily globally asymptotically stable.

1. Introduction and statement of the main result

For $n \geqslant 2$ let $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a C^1 vector field. As usual we will identify the vector field F with the ordinary differential equation

$$\dot{x} = \frac{dx}{dt} = F(x), \quad x = (x_1, \ldots, x_n), \tag{1}$$

where the dot denotes the derivative with respect to the independent variable t, called here the time.

Assume that $x^* \in \mathbb{R}^n$ is an equilibrium point of system (1). Assume further that there exists an open neighborhood U of x^* such that the orbits of (1) starting from U tend to x^* in forward time. The basin of attraction of x^* is the largest open set whose elements satisfy the above condition. The equilibrium point x^* of the vector field F is *globally asymptotically stable* if its basin of attraction is the whole $\mathbb{R}^n$. Thus, if F is globally asymptotically stable, no matter how far from the equilibrium point the initial condition is, the positive trajectory through it will converge to the equilibrium point.

The problem of determining the basin of attraction of an equilibrium point of a vector field is of great importance for applications of the stability theory of ordinary differential equations. Historically the global asymptotic stability problem is closely related to the Markus-Yamabe Conjecture [11].

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