ON THE MAXIMUM NUMBER OF LIMIT CYCLES OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS FORMED BY A CUBIC UNIFORM ISOCHRONOUS CENTER AND A LINEAR CENTER SEPARATED BY A STRAIGHT LINE

LOUIZA BAYMOUT[1], REBIHA BENTERKI[1] AND JAUME LLIBRE[2]

ABSTRACT. The determination of the maximum number of limit cycles and their possible positions in the plane is one of the most difficult problems in the qualitative theory of planar differential systems. This problem is related with the second part of the unsolved 16th Hilbert's problem. Due to their applications in modeling many natural phenomena the piecewise differential systems have recently attracted big attention. In general it is very difficult to determine the upper bound of the maximum number of limit cycles that a class of differential systems can exhibit. In this work we extend the second part of the 16th Hilbert's problem to the planar discontinuous piecewise differential systems formed by an arbitrary linear center and an arbitrary cubic uniform isochronous center separated by a straight line. We provide for this class of piecewise differential systems an upper bound of its maximal number of limit cycles.

1. Introduction and statement of the main results

Limit cycles are one of the main remarkable and important solutions of the differential equations. The notion of a limit cycle appeared firstly at the end of the 19th century with Poincaré [23]. Later on Hilbert stated a list of 23 problems for the advance of mathematical science, and from then it started an intensive research on these problems throughout the 20th century. From the 23 problems only the so-called 16th Hilbert's problem together with the Riemann conjecture remains open until now. The 16th Hilbert problem consists of two parts, the second part asks for an upper bound for the maximum number of limit cycles but only for planar polynomial differential systems of a given degree. For a differential system a *limit cycle* is an isolated periodic orbit in the set of all periodic orbits of this differential system.

Recently the second part of the 16th Hilbert's problem becomes an interesting topic of research for many scientists because of the main role of limit cycles in understanding and explaining the dynamics of many natural phenomena for example the Sel'kov model of glycolysis [24], that showed the existence of a stable limit cycle which represent the normal physiological behavior in the human body, also some non-linear electrical circuits exhibit limit cycle oscillations, which inspired the original Van der Pol model [26, 27], or one of the Belousov Zhavotinskii model [3], etc.

The dynamics of the piecewise differential systems appear frequently in many fields of applied mathematics, mechanics, electronics, economics, neuroscience, etc., see for instance [6, 21, 25], these systems become a very interesting topic. Research on piecewise linear discontinuous differential systems started with the studies of Andronov, Vitt and Khaikin about 1920 in [1]. Many studies on piecewise linear differential systems come from applications, as for instance control theory and electric circuit design. For discontinuous piecewise differential systems we can distinguish between two kinds of limit cycles, the sliding and the crossing ones. A *sliding limit cycle* is a limit cycle that contains some arc of the lines of discontinuity that separate the different differential systems which form the piecewise differential system.

2020 *Mathematics Subject Classification.* Primary 34C29, 34C25, 47H11.
Key words and phrases. Cubic uniform isochronous centers, linear centers, limit cycles, discontinuous piecewise differential systems.