GLOBAL NILPOTENT CENTERS OF CUBIC POLYNOMIAL HAMILTONIAN SYSTEMS SYMMETRIC WITH RESPECT TO THE y-AXIS

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Abstract. A global center for a vector field in the plane is a singular point p having $\mathbb{R}^2$ filled of periodic orbits with the exception of the singular point p. Polynomial differential systems of degree 2 have no global centers. In this paper we classify the global nilpotent centers of planar cubic polynomial Hamiltonian systems symmetric with respect to the y-axis.

1. Introduction and statement of the results

The notion of center goes back to Poincaré and Dulac, see [13, 4]. They defined a *center* for a vector field on the real plane as a singular point having a neighborhood filled of periodic orbits with the exception of the singular point. The problem of distinguishing when a singular point is a focus or a center, known as the focus-center problem started precisely with Poincaré and Dulac and is still active nowadays with many questions unsolved.

If an analytic system has a center, then it is known that after an affine change of variables and a rescaling of the time variable, it can be written in one of the following three forms:

$$\dot{x} = -y + P(x, y), \quad \dot{y} = x + Q(x, y),$$

called *linear type center*, which has a pair of purely imaginary eigenvalues,

$$\dot{x} = y + P(x, y), \quad \dot{y} = Q(x, y)$$

called *nilpotent center*, and

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y)$$

called *linearly zero center*, where $P(x, y)$ and $Q(x, y)$ in the previous three systems are real analytic functions without constant and linear terms defined in a neighborhood of the origin.

We recall that a *global center* for a vector field in the plane is a singular point p having $\mathbb{R}^2$ filled of periodic orbits with the exception of the singular

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