Liouvillian integrability of three dimensional vector fields

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November 30, 2023

Abstract

We consider a three dimensional complex polynomial, or rational, vector field (equivalently, a two-form in three variables) which admits a Liouvillian first integral. We prove that there exists a first integral whose differential is the product of a rational 1-form with a Darboux function, or there exists a Darboux Jacobi multiplier. Moreover, we prove that Liouvillian integrability always implies the existence of a first integral that is obtained by two successive integrations from a one-forms with coefficients in a finite algebraic extension of the rational function field.
MSC (2020): 34A99, 12H05, 34M15.