QUADRATIC DIFFERENTIAL SYSTEMS WITH A WEAK FOCUS OF FIRST ORDER AND A FINITE SADDLE-NODE

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Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers were written on these systems, a complete understanding of this class is still missing. Classical problems, and in particular, Hilbert's 16th problem, are still open for this class. One of the goals of recent researchers is the topological classification of quadratic systems. As this attempt is not possible in the whole class due to the large number of parameters (twelve, but, after affine transformations and time rescaling, we arrive at families with five parameters, which is still a large number), many subclasses are considered and studied. Specific characteristics are taken into account and this implies a decrease in the number of parameters, which makes possible the study.
In this article we perform a global study (modulo islands) of the class $\overline{\mathbf{Qwf1sn}}$ which is the closure within real quadratic differential systems, of the family **Qwf1sn** of all such systems which have a weak focus of first order and a finite saddle-node. The bifurcation diagram for this class, done in the adequate parameter space which is the 3-dimensional real projective space $\mathbb{RP}^3$, is quite rich in its complexity since yields 399 subsets with 192 topologically distinct phase portraits for $\overline{\mathbf{Qwf1sn}}$, 146 of which have a representative in **Qwf1sn**. It can be shown that some of these parts have at least two limit cycles.
The phase portraits are always represented in the Poincaré disc. The bifurcation set is formed by an algebraic set of bifurcations of singularities, finite or infinite and by a set of bifurcations which we suspect to be analytic corresponding to global separatrices which have connections, or double limit cycles. Algebraic invariants were needed to construct the algebraic part of the bifurcation set, symbolic computations to deal with some quite complex invariants and numerical calculations to determine the position of the analytic bifurcation set of connections.

Keywords: Quadratic differential systems; finite saddle-node; weak focus of first order; limit cycle; phase portraits; bifurcation diagram; algebraic invariants.