ON THE CROSSING LIMIT CYCLES CREATED BY DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEM FORMED BY THREE LINEAR HAMILTONIAN SADDLES

MARIA ELISA ANACLETO[1], JAUME LLIBRE[2], CLAUDIA VALLS[3] AND CLAUDIO VIDAL[1]

Abstract. We study planar discontinuous piecewise differential systems formed by three linear Hamiltonian saddles separated by the non-regular line $\Sigma = \{(x, y) \in \mathbb{R}^2 : (y = 0) \vee (x = 0 \wedge y \geq 0)\}$. We prove that when the linear Hamiltonian saddles are homogeneous they have no limit cycles, and when they are non homogeneous they can have at most three limit cycles having exactly one point on each branch of Σ, and at most one limit cycle having four intersection points on Σ. Moreover we show that they can have at most one limit cycle having four intersection points on Σ and three limit cycles having three intersection points on Σ, simultaneously. Thus we have solved the extension of the 16th Hilbert problem to this class of piecewise differential systems. Furthermore we show that for the three types of combination of the limit cycles here studied in two of them the upper bound are sharp by providing examples of these systems with the maximum number of possible limit cycles. For the remaining type of limit cycles the upper bound is four and we have examples with three limit cycles. So at this moment is an open problem to know if the sharp upper bound is four or three.

1. Introduction and statement of the main results

Discontinuous piecewise differential systems in the plane $\mathbb{R}^2$ formed by linear differential systems appeared in a natural way studying some mechanical systems, see the pioneering book [1]. Later on these kind of differential systems allowed to study several electrical circuits and many other natural phenomena, see for instance the books [5, 23], the survey [21], and the paper [24]. These differential systems also play a main role in the control theory, see for example the books [2, 9, 12, 13].

The dynamics of the discontinuous piecewise differential systems started to be well defined in the book [6].

One of the main objects in the qualitative theory of planar differential systems are the limit cycles. We recall that a *limit cycle* is an isolated periodic orbit in the set of all orbits of a given differential system.

We remark that, in general, it is not easy to provide an explicit upper bound for the maximum number of limit cycles in a given class of differential systems, and even harder is to prove that this bound is reached. Thus the famous 16th Hilbert

2010 *Mathematics Subject Classification.* 34C05, 34C07.
Key words and phrases. Limit cycles, linear Hamiltonian saddles, discontinuous piecewise differential systems, first integrals.