COEXISTENCE OF UNCOUNTABLY MANY ATTRACTING SETS FOR SKEW-PRODUCTS ON THE CYLINDER

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Abstract. The aim of this paper is to show that the existence of attracting sets for quasiperiodically forced systems can be extended to appropriate skew-products on the cylinder, homotopic to the identity, in such a way that the general system will have (at least) one attracting set corresponding to every irrational rotation number ρ in the rotation interval of the base map. This attracting set is a copy of the attracting set of the system quasiperiodically forced by a (rigid) rotation of angle ρ. This shows the co-existence of uncountably many attracting sets, one for each irrational in the rotation interval of the basis map.

1. Introduction

We want to show that the existence of attracting sets for quasiperiodically forced systems can be extended to a class of skew-products on the cylinder which are homotopic to the identity. These systems have an attracting set corresponding to every irrational rotation number ρ in the rotation interval of the base map. This attracting set is a copy of the attracting set of the system quasiperiodically forced by a (rigid) rotation of angle ρ. In particular we show that the systems from our class can have uncountably many coexisting attracting sets (one for each irrational in the rotation interval of the base map).

To better explain the above and to state the main result of the paper we need to recall the basics of rotation theory on the circle, define what we understand by an attracting set, and to fix some notation.

In what follows the circle $\mathbb{R}/\mathbb{Z}$ will be denoted by $\mathbb{S}^1$. To simplify the notation, given $x \in \mathbb{R}$, we will identify $[x] \in \mathbb{S}^1$ with its representative in $[0, 1)$ (that is, with the fractional part of x, denoted by $\{\{x\}\}$).

It is well known that there exists a natural projection $e\colon \mathbb{R} \longrightarrow \mathbb{S}^1$ defined by $e(x) := \{\{x\}\}$ and that any continuous circle map f lifts to a continuous map $F\colon \mathbb{R} \longrightarrow \mathbb{R}$ (called a *lifting of* f) in such a way that $f \circ e = e \circ F$. If F is a lifting of f, then $F + n$ is also a lifting of f for every $n \in \mathbb{Z}$ and there exists $d \in \mathbb{Z}$ such that $F(x + 1) = F(x) + d$ for every $x \in \mathbb{R}$. Such integer d is called the *degree of* f.

In this paper we are only interested in continuous degree one circle maps. These are continuous maps such that $F(x + 1) = F(x) + 1$ for every $x \in \mathbb{R}$ and every lifting F. We denote by $\mathfrak{L}_1$ the set of all liftings of continuous circle maps of degree one.

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