



## Phase Portraits of a Class of Continuous Piecewise Linear Differential Systems

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### Abstract

The phase portraits of the planar linear differential systems are very well known. This is not the case for the phase portraits of the planar continuous piecewise linear differential systems. In this paper we classify the phase portraits of the class of planar continuous piecewise linear differential systems of the form

$$\dot{x} = a|x| + by + c, \quad \dot{y} = \alpha|x| + \beta y + \gamma,$$

in the Poincaré disc when  $a\beta - b\alpha \neq 0$ , and prove the existence and uniqueness of limit cycles. Note that on the straight line  $x = 0$  these differential systems are only continuous.

**Keywords** Continuous piecewise linear differential system · Phase portrait · Limit cycle · Poincaré disc

**Mathematics Subject Classification** 34A36 · 34C07 · 37G05

### Introduction and Statement of the Main Result

Andronov et al. [1] started to study the piecewise linear differential systems in the 1920s for modeling some mechanical systems, but the interest on this kind of differential systems persists up to nowadays. During the past twenty years many authors studied the dynamics of the piecewise linear differential systems, which can model many problems of mechanics, electronics, economy more accurately, see for instance [3, 4, 14, 18].

While the phase portraits of the linear differential systems

$$\dot{x} = ax + by + c, \quad \dot{y} = \alpha x + \beta y + \gamma,$$

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