ON THE DYNAMICS OF A HYPERJEK MEMRISTIVE SYSTEM

JAUME LLIBRE[1] AND CLAUDIA VALLS[2]

Abstract. We study the existence of zero-Hopf bifurcations in the fourth order ordinary differential equation $\ddot{\ddot{x}} = -\dddot{x} - a\,\dddot{x} - b\dot{x}^2\,\dddot{x} - (1+x)\dot{x}$ called the hyperjerk memristive system. This system has a line filled with equilibria and it has a polynomial first integral H. Writing this equation as a first order differential system in $\mathbb{R}^4$ we prove that this system has a zero-Hopf equilibrium $(-1,0,0,0)$ and from it, bifurcate two cylinders filled with periodic orbits parameterized by the levels of the first integral. Moreover, the three-dimensional system obtained restricting the differential system in $\mathbb{R}^4$ to the invariant hypersurface $H = h$, exhibits two Hopf bifurcations producing periodic orbits in the center manifold of that restriction.

1. Introduction and statement of the main results

In [12] the authors introduced a cubic two-parametric fourth order differential equation system which generalizes the memristive system introduced in [2], which in its turn generalized the original definition of memristor given in [1]. This equation has a line of equilibria and it has hyperjerk dynamics for some values of the parameters, in the sense that it involves a fourth order differential equation. Moreover, it is chaotic for some values of the parameters and there exist trajectories starting from points in the unstable manifold in a neighborhood of an unstable equilibrium point [7]. Systems exhibiting this chaotic behaviour have attracted the interest of many authors see for instance [9, 10, 11]. From the pioneer work of Chua and Kang in [2] many researches have worked proposing different memristive systems with different applications in different areas depending on their properties and now it is a very active research subject mainly because of its applications, see for instance [3, 4, 6, 13, 14, 15, 17, 18, 19].

Writing this cubic two-parametric fourth order differential equation memristive system introduced in [12] as a first order differential system in $\mathbb{R}^4$ we get

$$\dot{x} = y, \quad \dot{y} = z, \quad \dot{z} = w, \quad \dot{w} = -y - z - aw - xy - bz^2w, \tag{1}$$

where a, b are real parameters.

2010 *Mathematics Subject Classification.* 34C05.
Key words and phrases. Hopf bifurcation, zero-Hopf bifurcation.