LIMIT CYCLES IN A CLASS OF PLANAR DISCONTINUOUS PIECEWISE QUADRATIC DIFFERENTIAL SYSTEMS WITH A NON-REGULAR DISCONTINUOUS BOUNDARY (I)

DONGPING HE[1] AND JAUME LLIBRE[2]

ABSTRACT. In this paper we study the limit cycles which bifurcate from the periodic orbits of the quadratic uniform isochronous center $\dot{x} = -y + xy$, $\dot{y} = x + y^2$, when this center is perturbed inside the class of all discontinuous piecewise quadratic polynomial differential systems in the plane with two pieces separated by a non-regular discontinuous boundary, which is formed by two rays starting from the origin and forming an angle $\alpha = \pi/2$. Using the averaging theory of first order and the Chebyshev theory we prove that the maximum number of hyperbolic limit cycles which can bifurcate from these periodic orbits is exactly 8. For this class of discontinuous piecewise differential systems we obtain three more limit cycles than if the discontinuous boundary is regular, i.e., the case of where these two rays form an angle $\alpha = \pi$.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

It is well-known that the second part of the *Hilbert's 16th problem* ask for an upper bound of the maximum number of limit cycles in function of the degree of the planar polynomial differential systems, and for the possible distributions of the limit cycles, see [15, 17, 19]. The possible distributions of limit cycles has been solved, see [25], but to find such upper bound remain unsolved. Therefore, more and more studies (see for example [2, 12, 19, 20] and the references therein) focus on the *weak Hilbert's 16th problem* proposed by Arnold [1], that concerns on the investigation of the maximum number of limit cycles bifurcating from the periodic orbits of the centers of polynomial differential systems when they are perturbed inside the class of all polynomial differential systems of degree n.

In this paper we study a particular case of the weak Hilbert's 16th problem, i.e. to find the maximum number of limit cycles bifurcating from the periodic orbits of a uniform isochronous center under discontinuous piecewise polynomial perturbations. Recall that for planar polynomial differential systems, Conti [9] proved that a center is called a *uniform isochronous center* if in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$ it can be written in the form $\dot{r} = R(\theta, r), \dot{\theta} = k$, where k is a nonzero real number.

It is easily to check that a linear differential system with a uniform isochronous center can be written as $\dot{x} = -y, \dot{y} = x$ through an affine change of variables and a rescaling of time. As it is indicated in [22] the quadratic polynomial differential system with a uniform isochronous center can be written into the following form,

$$\dot{x} = -y + xy, \quad \dot{y} = x + y^2. \tag{1}$$

In fact, this system, under the transformation $x = -Y, y = X$, can be transformed into the system (S_2) of Table 4 in [6], or of Table 1 in [27]. Therefore systems (1) and (S_2) are equivalent, and then

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