CHARACTERIZATION OF GLOBAL CENTERS BY THE MONODROMY AT INFINITY

ISAAC A. GARCÍA[1], JAUME GINÉ[2] AND JAUME LLIBRE[3]

Abstract. In this work we focus in the family of real planar polynomial vector fields of arbitrary degree. We are interested in to characterize when a (local) center singularity of these vector fields becomes a global center, that is, its period annulus foliates the punctured real plane. The characterization of any global center is done by blowing-down the polycycle at infinity into a monodromic singular point.

1. Introduction and statement of the main result

A *center* of a real planar polynomial vector field $\mathcal{X} = P(x,y)\partial_x + Q(x,y)\partial_x$, with $P, Q \in \mathbb{R}[x,y]$ polynomials of degree n, is an equilibrium point having a punctured neighborhood foliated by periodic orbits. A *global center* is a center p such that $\mathbb{R}^2 \backslash p$ is foliated by periodic orbits.

The notion of center goes back to the works of Huygens in 1656 about the pendulum clock, see [21, 28]. Some centuries later the definition of center was given in the works of Poincaré [29] in 1881 and Dulac [9] in 1908. To determine if a given differential system has a center at a singular point is in general a difficult problem, see for instance [12, 13, 18, 19] and references therein.

In general it is not easy to determine when a center is global. The method used up to know is based in the blow-up process [3], see for example [23,25]. However using the following result we propose a simple solution of the global center problem based in a well-known established algorithm for determining when a singular point is monodromic.

Theorem 1. *Let the origin be the unique singularity of a real planar polynomial vector field $\mathcal{X}$ of degree n. We consider the Bendixson compactification $\tilde{\mathcal{X}} = \phi_*(\mathcal{X})/(u^2+v^2)^n$ of $\mathcal{X}$ where ϕ_* is the pull-back*

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